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Retrieval properties of a neural network with an asymmetric learning rule

E Gardner[†], S Mertens[‡] and A Zippelius[‡]

[†] Department of Physics, University of Edinburgh, Edinburgh EH9 3JZ, UK

[‡] Institut für Theoretische Physik, Bunsenstrasse 9, D-3400 Göttingen, Federal Republic of Germany

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Abstract. We consider a Hebbian learning mechanism, which gives rise to a change in synaptic efficacies only if the postsynaptic neuron is active. The model is solved analytically in the limit of strong dilution. The network is shown to classify initial configurations according to their mean activity and their overlap with one of the learnt patterns. The capacity of the network is calculated as a function of threshold.

1. Introduction

In the last few years, spin-glass models have been used extensively to describe the collective properties of highly connected neural networks. The success of these models is partially due to the courageous simplifications which were introduced into the formulation of the model and which allow for a quantitative analysis by the methods of statistical mechanics. On the other hand it was realised from the very beginning that some of the assumptions made in the Hopfield-Little model [1] contradict physiology. Hence various attempts have been made to either lift these assumptions or otherwise introduce physiological constraints. We mention only a few such contradictory points.

(i) In biological networks the synaptic efficacies are asymmetric, if not unidirectional.

(ii) The connectivity in biological systems is high but not complete.

(iii) In the Hopfield-Little model all internal timescales are ignored, like presynaptic delays or postsynaptic summation times.

(iv) All synapses, which connect a given neuron to others, are either excitatory or inhibitory. Even though exceptions are known, this so-called Dale's law seems widely accepted for biological networks.

Asymmetry and dilution have been investigated by several groups [2, 3]; Dale's law has been implemented by Shinomoto [4] and various timescales have also been discussed [5]. One of the results of these studies is the enormous robustness of the Hopfield model to structural changes in the model. Asymmetries, dilution, delays etc do not affect the retrieval properties significantly and sometimes even improve the performance of the network.

In the above examples some firm knowledge about the underlying physiological processes is available. Much less is known about the long-term changes of neuronal connectivity, which provide a mechanism of learning. There is experimental evidence

(for a review see, e.g. [6])—mainly from the visual system—that the time-dependent correlations between the activity of the pre- and postsynaptic neuron play an important role in these long-term changes of the neuronal interactions. Furthermore some experiments [6] have led to the conclusion that it is the *activation of the postsynaptic neuron* which is essential for changes in the synaptic efficacies. Hebb [7] and others [8] suggested that the efficacy of an excitatory synapse should increase when both the pre- and the postsynaptic neuron are active, and that it should decrease when the postsynaptic neuron is active while the presynaptic neuron is silent. We have taken this learning rule seriously and studied the following model:

$$\Delta J_{ij} = \frac{1}{2}(\sigma_i + 1)\sigma_j. \quad (1)$$

The synaptic efficacy J_{ij} operating between the presynaptic neuron $\sigma_j = \pm 1$ and the postsynaptic neuron $\sigma_i = \pm 1$ is changed *only* if the postsynaptic neuron is active. An excitatory synapse is changed as suggested by Hebb, whereas inhibition becomes less effective if both neurons are simultaneously active and becomes more effective if the postsynaptic one is active and the presynaptic one is not.

2. The model

We consider a Hopfield-Little model for a network of N binary neurons $\sigma_i = \pm 1$, $i = 1, \dots, N$. Each active neuron $\sigma_j = 1$ contributes to the postsynaptic potential of neuron i

$$h_i(t) = \frac{1}{2} \sum_{j(\neq i)} J_{ij}(\sigma_j(t) + 1) \quad (2)$$

according to the synaptic strength J_{ij} . If the postsynaptic potential exceeds a threshold H_i then the postsynaptic neuron is activated

$$\sigma_i(t + \Delta t) = \text{sgn}(h_i(t) - H_0). \quad (3)$$

For simplicity we consider a uniform threshold only, $H_i = H_0 > 0$. We shall also consider noisy dynamics, for which the updating proceeds according to

$$\sigma_i(t + \Delta t) = \begin{cases} 1 & \text{with probability } \left[1 + \exp\left(-\frac{2h_i(t) - 2H_0}{T_0}\right) \right]^{-1} \\ -1 & \text{with probability } \left[1 + \exp\left(\frac{2h_i(t) - 2H_0}{T_0}\right) \right]^{-1}. \end{cases} \quad (4)$$

The parameter T_0 measures the strength of the noise and is therefore called ‘temperature’. We shall only consider parallel updating. It can, however, be shown [3] that the stationary states are the same for sequential updating.

The network has learnt p patterns according to the rule of (1), such that the resulting couplings are given by

$$\tilde{J}_{ij} := \frac{1}{2} \sum_{\mu=1}^p (\xi_i^\mu + 1)\xi_j^\mu \quad (5)$$

where ξ_i^μ denotes the value of site i in pattern μ . We assume the ξ_i^μ to be independent random numbers, which take on the values $\xi_i^\mu = \pm 1$ with equal probability. Note that the synaptic efficacies in (5) are not symmetric, because our learning rule differentiates

between post- and presynaptic neurons. Hence there is no *a priori* Liapunov functional, which decreases monotonically during the time evolution of the network.

Our model is most easily solved in the limit of strong dilution [3]: each synapse J_{ij} is present only with a small probability. This is achieved by replacing \tilde{J}_{ij} by

$$J_{ij} = C_{ij} \tilde{J}_{ij} \tag{6}$$

with $C_{ij} \in \{0, 1\}$ a random number, which is drawn from the distribution

$$\rho(C_{ij}) = \frac{C}{N} \delta(C_{ij} - 1) + \left(1 - \frac{C}{N}\right) \delta(C_{ij}). \tag{7}$$

The strong-dilution limit requires C fixed in the limit $N \rightarrow \infty$. Note that C_{ij} and C_{ji} are independent random numbers, so that the dilution also gives rise to asymmetry in the couplings J_{ij} .

In order to solve the model, we introduce two order parameters $p_1(t)$ and $p_2(t)$, giving

$$\sigma_i(t) = \begin{cases} \left. \begin{array}{l} \xi_i^1 \quad \text{with probability } p_1(t) \\ -\xi_i^1 \quad \text{with probability } (1 - p_1(t)) \end{array} \right\} & \text{for } \xi_i^1 = 1 \\ \left. \begin{array}{l} \xi_i^1 \quad \text{with probability } p_2(t) \\ -\xi_i^1 \quad \text{with probability } (1 - p_2(t)) \end{array} \right\} & \text{for } \xi_i^1 = -1. \end{cases} \tag{8}$$

The appearance of two order parameters is due to the fact that our model is not symmetric with respect to the up and down state (active and inactive neuron).

We define the overlap of the system with one pattern (e.g. the first) $q(t)$ and the magnetisation (or mean activity) $m(t)$ as

$$q(t) := \frac{1}{N} \sum_{i=1}^N \langle \xi_i^1 \sigma_i(t) \rangle \tag{9}$$

$$m(t) := \frac{1}{N} \sum_{i=1}^N \langle \sigma_i(t) \rangle \tag{10}$$

where $\langle \cdot \rangle$ denotes the average over the $\{C_{ij}\}$, the patterns and the stochastic dynamics. The overlap and the magnetisation are related to p_1 and p_2 according to

$$m(t) = p_1(t) - p_2(t) \tag{11}$$

and

$$q(t) = p_1(t) + p_2(t) - 1. \tag{12}$$

The dynamic evolution of these quantities is given through two coupled equations of motion, as shown in the appendix. These equations simplify considerably in the limit $C \rightarrow \infty$ (after performing the limit $N \rightarrow \infty$ such that $C/N \rightarrow 0$):

$$m(t + \Delta t) = \frac{1}{\sqrt{4\pi}} \int_{-\infty}^{\infty} dy \exp(-y^2) \left[\tanh\left(\frac{q(t) - \sqrt{2\alpha(1+m(t))}y - 2H}{2T}\right) - \tanh\left(\frac{\sqrt{2\alpha(1+m(t))}y + 2H}{2T}\right) \right] \tag{13}$$

and

$$q(t + \Delta t) = \frac{1}{\sqrt{4\pi}} \int_{-\infty}^{\infty} dy \exp(-y^2) \left[\tanh\left(\frac{q(t) - \sqrt{2\alpha(1+m(t))}y - 2H}{2T}\right) - \tanh\left(\frac{\sqrt{2\alpha(1+m(t))}y - 2H}{2T}\right) \right]. \tag{14}$$

Here we have introduced a ‘reduced’ temperature T and a ‘reduced’ threshold H

$$T := T_0/C \quad H := H_0/C$$

and α , given by

$$\alpha := (P - 1)/C \tag{15}$$

denotes the number of stored patterns with respect to the existing couplings.

In the following sections we shall discuss the stationary solutions of the equations of motion (equations (13) and (14)).

3. The case $\alpha = 0$

The retrieval properties are most easily analysed if the number of patterns is negligible as compared to the mean connectivity C , i.e. $\alpha = 0$. In this limit the fixed-point equation for the overlap reads

$$q = \frac{1}{2} \tanh\left(\frac{q - 2H}{2T}\right) + \frac{1}{2} \tanh\left(\frac{H}{T}\right). \tag{16}$$

Zero overlap $q^* = 0$ is always a solution of (16); it is stable for sufficiently large fields

$$\cosh^2(H/T) > 1/4T. \tag{17}$$

The line of instability $H(T)$ or $T(H)$ defined by (17) is shown in figure 1 as $T_s(H)$. To the left of this line $q^* = 0$ is unstable and there are two stable fixed points—one

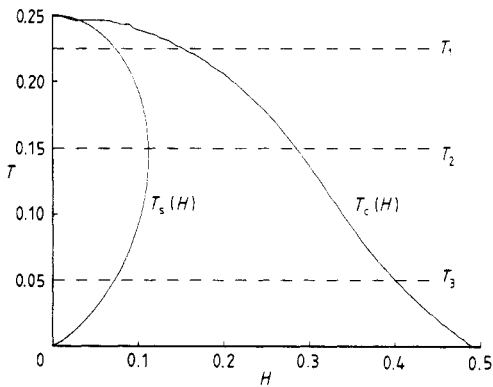


Figure 1. Phase diagram for $\alpha = 0$.

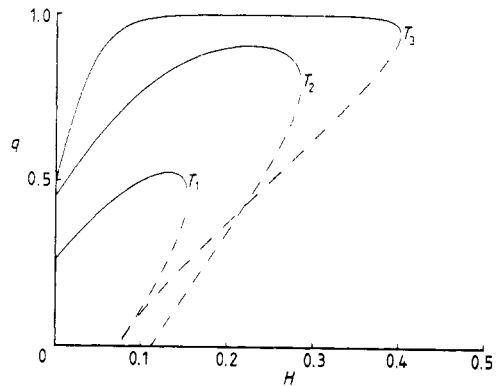


Figure 2. Stationary (full curve) and critical (broken curve) overlap as a function of H for the three temperatures indicated in figure 1. The dotted line indicates the limit $T \rightarrow 0$.

with positive and one with negative overlap. To the right of this line $q^* = 0$ is a stable fixed point and all initial configurations with a negative overlap are attracted to it. In addition to the zero-overlap solution, there is a pair of fixed points with positive overlap, one stable, q^* , and one unstable, q_c , such that $0 \leq q_c \leq q^*$. Hence there is a critical overlap $q_c(H)$, such that an initial configuration with $q > q_c$ is recognised (attracted to the fixed point q^*), whereas an initial overlap with $q < q_c$ is iterated towards zero overlap. The fixed point $q^*(H) > 0$ persists up to a critical temperature $T_c(H)$, where the unstable (q_c) and stable (q^*) fixed points merge. No retrieval is possible beyond $T_c(H)$. The overlap q^* jumps discontinuously to a finite value, when the line $T_c(H)$ is approached from high temperatures or large fields. This is most easily seen for $T = 0$, where (16) becomes

$$q = \frac{1}{2}(\text{sgn}(q/2 - H) + \text{sgn} H). \tag{18}$$

At a critical field $H = \frac{1}{2}$ the overlap jumps discontinuously from zero to one. In figure 2 we show the fixed-point value q^* together with the critical overlap q_c as a function of H for various temperatures.

The magnetisation

$$m = \frac{1}{2} \tanh\left(\frac{q - 2H}{2T}\right) - \frac{1}{2} \tanh\left(\frac{H}{T}\right) \tag{19}$$

is slaved to q : it is positive for $q > 4H$ and negative otherwise.

4. The case $T = 0$

We now have to look for fixed points in the (q, m) plane, which are determined by the coupled equations for the overlap q and the magnetisation m

$$q = \frac{1}{2} \left\{ \text{erf}\left(\frac{q - 2H}{\sqrt{2\alpha(1+m)}}\right) + \text{erf}\left(\frac{2H}{\sqrt{2\alpha(1+m)}}\right) \right\} =: g(q, m) \tag{20}$$

$$m = \frac{1}{2} \left\{ \text{erf}\left(\frac{q - 2H}{\sqrt{2\alpha(1+m)}}\right) - \text{erf}\left(\frac{2H}{\sqrt{2\alpha(1+m)}}\right) \right\} =: f(q, m). \tag{21}$$

The ferromagnetically aligned state ($q = 0, m = -1$) is a stable fixed point for all values of α and H . Since it has no correlation with any of the learnt patterns, it can be regarded as the ‘waste basket’ for those initial configurations of the network which cannot be classified according to their correlation with the learnt patterns.

Retrieval states with a finite overlap $q \neq 0$ with one pattern exist for sufficiently small H and α , as shown in figure 3(a). There is a continuous transition for $H < H_M$, such that q^* goes continuously to zero as one approaches $\alpha_1(H)$ from below. The line $\alpha_1(H)$ coincides with the limit of stability of $q^* = 0$ and is given by

$$\sqrt{2\pi\alpha_1(1+m)} = \exp\left(-\frac{2H^2}{\alpha_1(1+m)}\right). \tag{22}$$

For $H > H_M$, there is a discontinuous transition such that q^* jumps to a finite value when the line $\alpha_2(H)$ is crossed from the side of large α or H . The line $\alpha_2(H)$ ends in the point $(\alpha = 0, H = \frac{1}{2})$ in agreement with the results of § 3. Within the retrieval phase (the region under the lines $\alpha_1(H)$ and $\alpha_2(H)$ in figure 3(a)) there is another line $\alpha_3(H)$, such that to the left of this line there is also a stable fixed-point solution

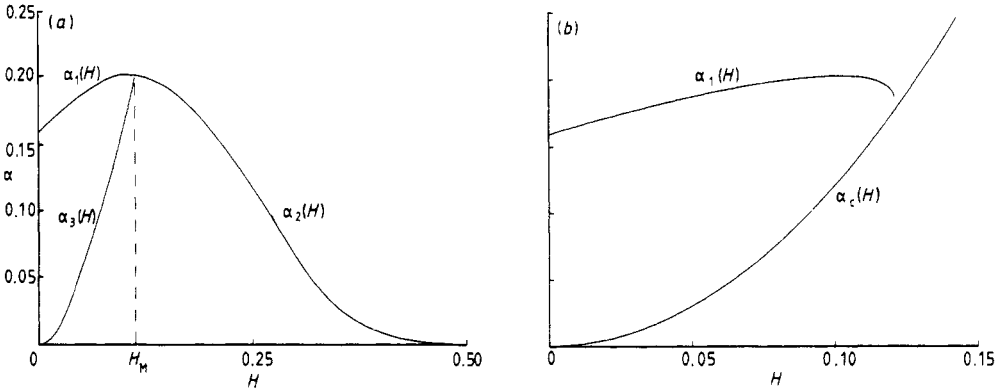


Figure 3. (a) Phase boundaries for the retrieval phase. (b) Limit of stability $\alpha_1(H)$ and existence $\alpha_c(H)$ of the partially aligned state with zero overlap.

with a negative overlap $q^* < 0$, which attracts initial conditions which are anticorrelated with one of the patterns. To the right of the line $\alpha_3(H)$ no such solution exists. The magnetisation in the retrieval state $q^* > 0$ can be either positive or negative, depending on the parameters α and H .

Besides the ferromagnetic state and the retrieval state, there can be another fixed point with zero overlap and imperfect alignment ($m > -1$). It is stable provided

$$1 - \left. \frac{\partial g}{\partial q} \right|_{q=0, m^*} > 0 \tag{23}$$

and

$$1 - \left. \frac{\partial f}{\partial m} \right|_{q=0, m^*} > 0 \tag{24}$$

where m^* has to be determined self-consistently from

$$m^* = -\operatorname{erf}\left(\sqrt{\frac{2H^2}{\alpha(1+m^*)}}\right). \tag{25}$$

This latter equation always has the stable solution $m^* = -1$, as claimed above. For sufficiently large $\alpha > \alpha_c \approx 12.07 H^2$ there is another pair of solutions, one stable (m_s) and one unstable (m_c) with respect to fluctuations in m . These are shown in figure 4. The parabola $\alpha_c = 12.07 H^2$ defines a line in the (α, H) plane, as shown in figure 3(b). This line coincides with the limit of stability determined by (24)

$$1 = \sqrt{\frac{2H^2}{\alpha\pi}} \frac{1}{(1+m^*)^{3/2}} \exp\left(-\frac{2H^2}{\alpha(1+m^*)}\right). \tag{26}$$

Stability with respect to fluctuations in q (23) requires

$$\sqrt{2\pi\alpha(1+m^*)} > \exp\left(-\frac{2H^2}{\alpha(1+m^*)}\right). \tag{27}$$

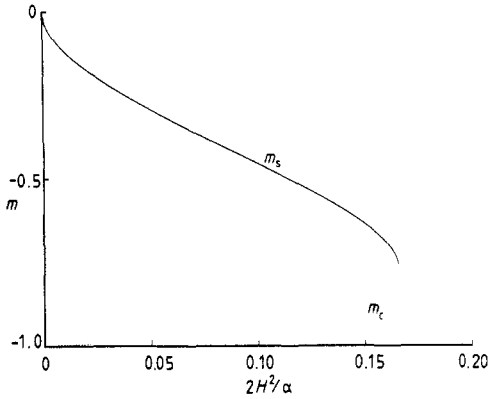


Figure 4. Magnetisation in the retrieval phase with zero overlap.

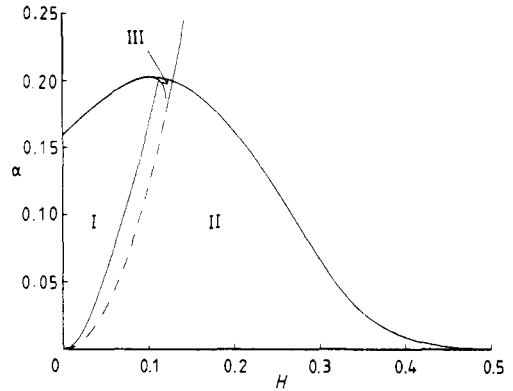


Figure 5. Figures 3(a) and (b) superimposed, giving rise to three different retrieval phases (I, II, III) in the (α, H) plane.

Note that the basin of attraction of the ferromagnetic state must be rather small, if the unstable fixed point exists, as can be seen from figure 4.

These results are put together in figure 5. In regions I, II and III the network is able to retrieve a learnt pattern. Region I is characterised by two stable fixed points, one with positive and one with negative overlap (besides the ferromagnetic state, which is stable in the whole (α, H) plane). In figure 6(a) we have indicated their basins of attraction. The points in the (q, m) plane denote an initial condition and the symbols associated with the points denote the asymptotic states which are reached by iteration of (20) and (21). Note that the magnitude of m and q is restricted by $|m \pm q| \leq 1$. In region II there is only one fixed point with positive overlap and in region III there

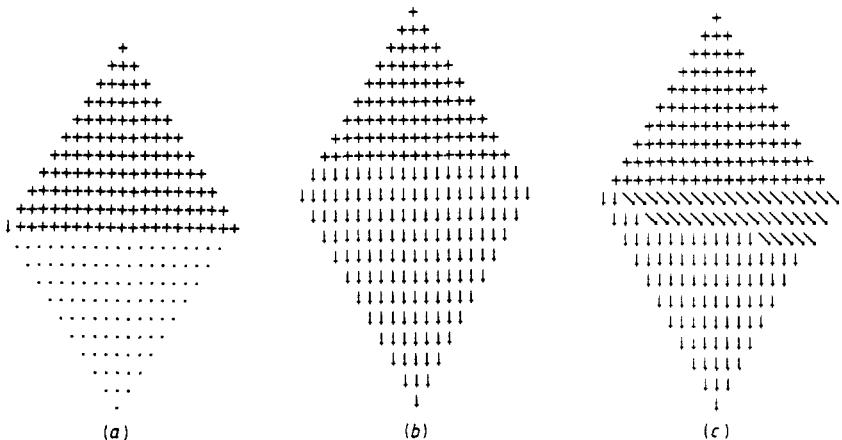


Figure 6. Basins of attraction in the m (horizontal axis) and q (vertical axis) plane for the three retrieval phases of figure 5. They correspond to (a) $(\alpha, H) = (0.1, 0.05)$ (phase I), (b) $(\alpha, H) = (0.1, 0.15)$ (phase II) and (c) $(\alpha, H) = (0.197, 0.1205)$ (phase III). The symbols denote:

- \downarrow fully aligned ($m = -1, q = 0$)
- \cdot partially aligned ($-1 < m < 0, q = 0$)
- $+$ $q > 0$
- $-$ $q < 0$.

are two fixed points, one with positive and another one with zero overlap. Their basins of attraction are shown in figures 6(b) and 6(c).

5. Conclusions

We have studied a Hopfield network with a learning rule, which is based on the assumption that activation of the postsynaptic neuron is required for long-term modification of synaptic efficacies. The stationary states of the model have been analysed and the phase diagram has been calculated as a function of threshold field H and capacity α . Several different retrieval phases were shown to exist (see figure 5). The network can serve as an associative memory: it classifies an initial configuration according to its mean activity and its overlap with one of the learnt patterns. In particular in region II all those configurations whose initial overlap is too small are attracted to a state with no correlation with any of the learnt patterns. This so-called ‘waste basket’ is the completely inactive state. The capacity of the network is found to depend on the threshold of the postsynaptic neuron; it is optimal for $H \approx 0.1$, and no retrieval is possible for $H > \frac{1}{2}$.

It would be interesting to study the timescale of relaxation into a retrieval state, as well as the detailed structure of the attractors. In biological networks there seems to be no evidence for interactions, which are independent of the state of the neurons. Hence it might be useful to study a wider class of models with interactions which are not symmetric with respect to the up and down (active and inactive) state. Work along this line is in progress.

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Appendix

The basic idea which enables the derivation of the equations of motion is the observation that in the ‘strong dilution limit’ the dynamic ‘ancestors’ of a neuron σ_i (i.e. those neurons σ_j , for which $J_{ij} \neq 0$) are mutually correlated only with probability zero. Details of this idea can be found in [3].

We label the dynamic ‘ancestors’ of neuron σ_i with $\sigma_{j_1}, \dots, \sigma_{j_K}$ and get for the local field

$$h_i(t) = \frac{1}{2}(1 + \xi_i^1) \sum_{r=1}^K \xi_{j_r}^1 \frac{1}{2}(\sigma_{j_r}(t) + 1) + \sum_{\mu=2}^P \sum_{r=1}^K \frac{1}{2}(\xi_r^\mu + 1) \xi_{j_r}^\mu \frac{1}{2}(\sigma_{j_r}(t) + 1) - H_0$$

where we have included the threshold into the definition of h_i . We define

$$\tau_{j_r} := \frac{1}{2} \xi_{j_r}^1 (\sigma_{j_r}(t) + 1) \quad (28)$$

and obtain

$$\tau_{j_r} = \begin{cases} 0 & \text{with probability } \frac{1}{2}(1 - p_1(t) + p_2(t)) \\ 1 & \text{with probability } \frac{1}{2}p_1(t) \\ -1 & \text{with probability } \frac{1}{2}(1 - p_2(t)). \end{cases} \quad (29)$$

If we denote the number of occurrences of 0 and of -1 in the set τ_j , with l_0 (and respectively l_1), we have

$$\sum_{r=1}^k \tau_j = k - l_0 - 2l_1$$

with probability

$$\frac{k!}{l_0!l_1![k-l_0-l_1]} \frac{(1-p_1(t)+p_2(t))^{l_0}(1-p_2(t))^{l_1}p_1(t)^{k-l_0-l_1}}{2^k}. \quad (30)$$

Due to the assumption that the state of the network is uncorrelated to all patterns but the first, we get for $\mu > 1$

$$\frac{1}{2}(\xi_i^\mu + 1)\xi_{j_r}^\mu \frac{1}{2}(\sigma_{j_r}(t) + 1) = \begin{cases} 0 & \text{with probability } \frac{1}{4}(3-p_1(t)+p_2(t)) \\ 1 & \text{with probability } \frac{1}{8}(1+p_1(t)-p_2(t)) \\ -1 & \text{with probability } \frac{1}{8}(1+p_1(t)-p_2(t)). \end{cases} \quad (31)$$

We denote the numbers of occurrences of 0 and of (-1) in the set

$$\left\{ \frac{1}{2}(\xi_i^\mu + 1)\xi_{j_r}^\mu \frac{1}{2}(\sigma_{j_r}(t) + 1) \right\}_{\mu=2 \dots k}^{r=1 \dots P}$$

with n_0 (and respectively n_1) and get

$$\sum_{\mu=2}^P \sum_{r=1}^k \frac{1}{2}(\xi_i^\mu + 1)\xi_{j_r}^\mu \frac{1}{2}(\sigma_{j_r}(t) + 1) = k(P-1) - n_0 - 2n_1$$

with probability

$$\frac{[k(P-1)]!}{n_0!n_1![k(P-1)-n_0-n_1]} \frac{(3-p_1(t)+p_2(t))^{n_0}(1+p_1(t)-p_2(t))^{k(P-1)-n_0}}{2^{3k(P-1)-n_0}}. \quad (32)$$

Combining (30), (32) and (34) we obtain

$$\begin{aligned} m(t + \Delta t) &= \frac{1}{N} \sum_{i=1}^N \left\langle \tanh \frac{h_i(t)}{T_0} \right\rangle \\ &= \Sigma(C, P; p_1(t), p_2(t)) \circ \frac{1}{2} \left[\tanh \left(\frac{k(P-1) - n_0 - 2n_1 - H_0}{T_0} \right) \right. \\ &\quad \left. + \tanh \left(\frac{k - l_0 - 2l_1 + k(P-1) - n_0 - 2n_1 - H_0}{T_0} \right) \right] \end{aligned} \quad (33)$$

where $\Sigma(C, P; p, q)$ is a linear operator which maps a function $f(k, l_0, l_1, n_0, n_1)$ onto the real axis via

$$\Sigma(C, P; p_1, p_2) \circ f$$

$$\begin{aligned} &:= \sum_{k=0}^{\infty} \frac{C^k}{k!} e^{-C} \sum_{l_0=0}^k \sum_{l_1=0}^{k-l_0} \frac{k!}{l_0!l_1![k-l_0-l_1]} \\ &\quad \times \frac{(1-p_1(t)+p_2(t))^{l_0}(1-p_2(t))^{l_1}p_1(t)^{k-l_0-l_1}}{2^k} \\ &\quad \times \sum_{n_0=0}^{k(P-1)} \sum_{n_1=0}^{k(P-1)-n_0} \frac{[k(P-1)]!}{n_0!n_1![k(P-1)-n_0-n_1]} \\ &\quad \times \frac{(3-p_1(t)+p_2(t))^{n_0}(1+p_1(t)-p_2(t))^{k(P-1)-n_0}}{2^{3k(P-1)-n_0}} f(k, l_0, l_1, n_0, n_1). \end{aligned}$$

We have used the fact that a neuron has k dynamic ancestors with probability

$$w(k) = \binom{N}{k} \left[\frac{C}{N} \right]^k \left[1 - \frac{C}{N} \right]^{N-k} \\ \approx \frac{C^k}{k!} \exp(-C) \quad (34)$$

where the second line holds in the 'strong dilution limit'.

An analogous consideration yields the equation of motion for q :

$$q(t + \Delta t) = \Sigma(C, P; p_1(t), p_2(t)) \circ \frac{1}{2} \left[\tanh \left(\frac{k(P-1) - n_0 - 2n_1 - H_0}{T_0} \right) \right. \\ \left. + \tanh \left(\frac{k - l_0 - 2l_1 + k(P-1) - n_0 - 2n_1 + H_0}{T_0} \right) \right]. \quad (35)$$

To discuss the equations of motion for $m(t)$ and $q(t)$ it is convenient to allow for a large connectivity C . This does not spoil our argument concerning the uncorrelated dynamic ancestors since we perform the limit $N \rightarrow \infty$ first. We therefore consider the limit $C \rightarrow \infty$, $P \rightarrow \infty$ such that

$$\alpha := (P-1)/C$$

is finite. $\alpha > 0$ means that we have a macroscopic number of stored patterns (with respect to the number of bonds). It is a straightforward calculation to show that in this limit (33) and (35) lead to (13) and (14).

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